



Reg. No. :

Name :

Fifth Semester B.Tech. Degree Examination, November 2013
(2008 Scheme)

08.501 : ENGINEERING MATHEMATICS – IV (ERFBH)

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries 4 marks.

1. The following is the p.d.f. of a random variable X :

x	:	0	1	3	7	13
P(X = x)	:	k	2k	2k	3k	k ²

Find k and mean of X.



- The number of hits to a popular website during a 1-minute interval is a Poisson random variable with mean 2.4. The server hosting the page is configured to handle a maximum of 4 requests per minute. What is the probability that the server will receive more than 4 request would arrive in a 1 minute period.
- 1000 light bulbs with mean length of life 120 days are installed in a factory. Their length of life is assumed to follow normal distribution with S.D. 20 days. If it is decided to replace all the bulbs together, what interval should be allowed between replacements if not more than 10% should expire before replacement ?
- If a random variable X is uniformly distributed over $(-\alpha, \alpha)$, find α so that $P(|X| < 1) = P(|X| > 1)$.
- The mean of a sample of size 20 from a normal population with S.D. 8 was found to be 81.2. Find a 90% confidence interval for the population mean.
- Can $y = 2.8x + 5$ and $x = 3 - 0.5y$ be the estimated regression equation of Y on X and X on Y respectively ? Explain your answer.
- In a city 325 men out of 600 were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers ?
- Define wide sense stationary and strict sense stationary processes. Show that every strict sense stationary process is also wide sense stationary.



9. Explain how the auto correlation function, power and power spectral density of a wide sense stationary process are related to one another.
10. Is a Poisson process stationary in any sense ? Justify your answer.

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

MODULE – I

11. a) In testing a certain kind of truck tire over a rugged terrain, it is found that 25% of the trucks fail to complete the test run without a blowout. Of the next 15 trucks tested, find the probability that i) from 3 to 5 blowouts ; ii) fewer than 4 have blowouts ; iii) more than 5 have blowouts.
- b) The spot speeds at a particular location are normally distributed with a mean of 51.7 km/hour and a standard deviation of 8.3 km/hour. What is the probability that i) the speed exceeds 65 km/hour ? ii) and the speed lie between 40 km/hour and 70 km/hour ? iii) What is the 85th percentile speed ?
- c) The time that a machine will run without repair is exponentially distributed with mean 200 days. Find the probability that such a machine will i) have to repair in less than 100 days, ii) not have to repair in atleast 250 days.
12. a) If $f(x) = \begin{cases} \frac{x+1}{2} & \text{for } |x| \leq 1 \\ 0 & \text{elsewhere} \end{cases}$ is the p.d.f. of X, find the mean and variance.
- b) The marks obtained by a batch of students in Mathematics are approximately normally distributed with mean 60 and standard deviation 5. If 5 students are selected at random from this group, what is the probability that atleast two of them will score above 80 ?
- c) Buses arrived at a specified stop at 15 minute intervals starting at 7 am. A passenger arrives at the stop at random time between 7 and 7.30 am. Find the probability that he waits i) less than 5 minutes, ii) atleast 12 minutes.



MODULE – II



13. a) Calculate the correlation coefficient for the following data

x : 65 66 67 67 68 69 70 72
y : 67 68 65 68 72 72 69 71

- b) A certain injection administered to each of 12 patients resulted in the following increases of blood pressure 5, 2, 8, - 1, 3, 0, 6, - 2, 1, 5, 0, 4. Can it be concluded that the injection, in general, accompanied by an increase in blood pressure. (Use 5% level of significance).
- c) In two colleges affiliated to a university 46 out of 200 and 48 out of 250 candidates failed in an examination. If the percentage of failure in the university is 18%, examine whether the colleges differ significantly.

14. a) Using the principle of least squares, fit a parabola of the form $y = ax^2 + bx + c$ for the following data :

X : 0 1 2 3 4 5
Y : 14 18 22 27 38 40

b) Two types of batteries are tested for their length of life and the following results were obtained :

	No. of Sample	Mean (Hrs.)	Variance
Battery A	10	500	100
Battery B	15	560	121

Is there a significance difference in the two means ?

c) The mean breaking strength of the cables supplied by a manufacturer is 1800 with SD of 100. By a new technique in the manufacturing process, it is claimed that the braking strength of the cable has increased, to test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850. Can we support the claim at 1% level of significance assuming that the S.D. has not changed ?



MODULE – III

15. a) The joint distribution of a two-dimensional random variable (X, Y) is given by $p(x, y) = c(2x + 3y)$, $x = 0, 1, 2$, $y = 1, 2, 3$. Find i) the value of c ii) the marginal distributions. Also find the probability distribution of $X + Y$.
- b) A random experiment consists of tossing a fair die repeatedly and let $X(n)$ denote the number that turns up at the n^{th} toss. Find the mean and autocorrelation of the random process $X(n)$. Is the process stationary ?
- c) $X(t)$ and $Y(t)$ are independent, zero mean, wide sense stationary processes and $Z(t) = X(t) + Y(t)$. Show that i) $R_{ZZ}(\tau) = R_{XX}(\tau) + R_{YY}(\tau)$ and ii) $S_{ZZ}(\omega) = S_{XX}(\omega) + S_{YY}(\omega)$.
16. a) A particular hospital keeps records of the arrival of patients in the emergency room. Those records show that, the number of arriving patients constitutes a Poisson process with an average rate of 6 patients per hour. For a given day, starting at 12:00 midnight,
- Find the probability that exactly 15 patients arrive between 12:00 midnight and 2:30 A.M.
 - What is the expected number of patients to arrive in one day ?
 - Find the probability that the first patient does not arrive until 12:30 A.M.
 - What is the expected time until the first patient arrives ?
- b) It is observed that customers of three different brands (A, B and C) of toothpastes have a tendency to switch brands in their next purchase. The following transition probability matrix shows the probability that a customer may or may not change brand in a subsequent purchase.

$$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$$

- Find the proportion of the customers who prefer each of the brands, A, B and C.
- A survey conducted among selected customers showed that 30% of them will go for brand A, while 20% will go for brand B and 50% will go for brand C in the current purchase. What will be the probability distribution of the preferences of these selected customers for the various brands at the second purchase after the current ?